

# TRIGONOMETRY LAWS AND IDENTITIES

## TANGENT IDENTITIES

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

## RECIPROCAL IDENTITIES

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

## PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

## PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta$$

$$\tan(\theta + \pi n) = \tan \theta$$

$$\cot(\theta + \pi n) = \cot \theta$$

## EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

## DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

## HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

## LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

## LAW OF SINES

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

## LAW OF TANGENTS

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

## PRODUCT TO SUM IDENTITIES

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

## SUM TO PRODUCT IDENTITIES

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

## SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

## MOLLWEIDE'S FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

## COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

## Trigonometric series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad [|x| < \infty]$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad [|x| < \infty]$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \dots \quad [|x| < \pi/2]$$

## Inverse trigonometric series

$$\arcsin x = x + \frac{x^3}{2.3} + \frac{1.3}{2.4.5}x^5 + \frac{1.3.5}{2.4.6.7}x^7 + \dots \quad [|x| < 1]$$

$$\arccos x = \frac{\pi}{2} - \arcsin x \quad [|x| < 1, 0 < \arccos x < \pi]$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad [|x| < 1]$$

## INVERSE TRIG FUNCTION NOTATION

$$\sin^{-1} x \equiv \arcsin x \equiv \text{Asin } x$$

$$\cos^{-1} x \equiv \arccos x \equiv \text{Acos } x$$

$$\tan^{-1} x \equiv \arctan x \equiv \text{Atan } x$$

## INVERSE TRIG DOMAIN

$$\sin^{-1} x : -1 \leq x \leq 1$$

$$\cos^{-1} x : -1 \leq x \leq 1$$

$$\tan^{-1} x : -\infty \leq x \leq \infty$$

